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Supersymmetric Models With $\tan\beta$ Close to Unity

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ABSTRACT

Within the framework of supersymmetric grand unification, estimates of the b quark mass based on the asymptotic relation $m_b \simeq m_\tau$ single out the region with $\tan\beta$ close to unity, particularly if $m_t(m_t) \lesssim 170 \text{ GeV}$. We explore the radiative breaking of the electroweak symmetry and the associated sparticle and higgs spectroscopy in models with $1 < \tan\beta \lesssim 1.6$. The lightest scalar higgs is expected to have a mass below 100 GeV , while the remaining four higgs masses exceed 300 GeV . The lower bounds on some of the sparticle masses are within the range of LEP 200.

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Recent improvements in the determination of the three standard model gauge couplings has renewed interest in the exciting possibility that they do indeed converge to the same value at some superheavy scale $\sim 10^{16} \text{ GeV}$ [1]. Within the framework of standard grand unification (GUT), compatibility with the available data requires an intermediate mass scale, on the order of $10^{12\pm 1} \text{ GeV}$ in some schemes. Perhaps a somewhat more elegant possibility is provided by supersymmetric grand unification (SUSY GUT), with SUSY broken at a scale $M_S \sim 100 \text{ GeV}$ to a few TeV. Among other things, this latter approach offers the prospects of resolving the formidable gauge hierarchy problem[2].

One could also hope that SUSY GUTs can shed light on at least some of the many parameters appearing in the minimal supersymmetric $SU(3) \times SU(2) \times U(1)$ model (MSSM). A particularly important example is offered by the well known asymptotic relation $m_b^0 \approx m_\tau^0$ [3]. It has become clear in recent years[4] that the determination of the b -quark mass from this relation depends in a rather interesting way on $m_t(m_t)$, $\alpha_S(M_Z)$ and $\tan \beta$. In particular, and we verify this in the first part of the paper, for $\alpha_S(M_Z)$ near 0.12 and $m_t(m_t) \lesssim 170 \text{ GeV}$, the two regions, $\tan \beta$ close to unity ($1 \lesssim \tan \beta \lesssim 1.6$), and $\tan \beta \gg 1$ are singled out. The large $\tan \beta$ case can arise naturally in minimal $SO(10)$ type GUTs[5] and has been the subject of several investigations[6]. However, weak-scale radiative corrections to the b mass through gluino exchange[7] can be quite substantial in this case. With $\tan \beta$ near unity and a top mass in the range of 150 - 170 GeV (favored by precision electroweak measurements), the top-quark Yukawa coupling h_t is near the infrared quasi-fixed point $h_t(m_t) \simeq 1.1$ [8]. This may be an appealing feature in that a wide range of initial values $h_t(M_X)$ near the GUT scale will be mapped to essentially the same value of $h_t(m_t)$ thereby making it insensitive to the initial conditions. Furthermore, with $\tan \beta$ close to unity, the weak-scale radiative corrections to the b mass can be quite small ($\lesssim 1 - 2\%$).

Some investigations have recently appeared in which $\tan \beta$ close to unity has been considered, although not exclusively so [9]. In this paper we primarily focus on this possibility and, where appropriate, offer comparisons with the large $\tan \beta$ case. It turns

out that the low $\tan\beta$ case is considerably less restrictive as far as the allowed parameter space is concerned. In particular, some of the sparticles including a stau and stop may well be accessible to LEP 200. With $m_t(m_t) \lesssim 170 \text{ GeV}$ the lightest CP even scalar higgs mass is expected to be $\lesssim 100 \text{ GeV}$.

The starting point of our computations are the two-loop renormalization group (RG) equations for the gauge and Yukawa couplings [10]. For simplicity, we will only consider two possible values for the (common) supersymmetry threshold M_S , a ‘low’ value comparable to m_t , and a ‘high value’ of order a TeV . Using $\alpha(M_Z) = 1/128$ and $\sin^2\theta_W(M_Z) = 0.233$ as inputs, we estimate the unification scale M_X to be about $2 \times 10^{16} \text{ GeV}$, while the unified gauge coupling $\alpha_G(M_X) \approx 1/26$.

The running b quark mass has been estimated to be $m_b(m_b) = 4.25 \pm 0.1 \text{ GeV}$ [11]. Considering the various hadronic uncertainties that are inherent in this determination, we will allow for a more generous 2σ error and take $m_b(m_b)$ to lie in the mass range $4.05 - 4.45 \text{ GeV}$. The upper end will turn out to be crucial in constraining $\tan\beta$. It corresponds to a physical mass (pole mass) $M_b = 5.2 \text{ GeV}$ (5.1 GeV) for $\alpha_s(M_Z) = 0.12$ (0.115). A value of M_b much larger than this seems unlikely.

The simplest (minimal) GUTs based on $SU(5)$ or $SO(10)$ predict the asymptotic relation $m_b^0 = m_\tau^0$, which holds above the unification scale $M_X \simeq 2 \times 10^{16} \text{ GeV}$. Deviation from the minimal GUT schemes, it appears, are necessary in order to obtain realistic quark and lepton masses. Even then, the above relation can hold to a good approximation. For instance, implementation of the Fritzsch ansatz in GUTs requires a non-minimal Higgs system [12]. From the asymptotic relation $Tr(M_\ell) = Tr(M_d)$,

$$m_d^0 - m_s^0 + m_b^0 = m_e^0 - m_\mu^0 + m_\tau^0 \quad (1)$$

Equation (6) leads to $m_b^0 = m_\tau^0(1 - \epsilon)$, where $\epsilon \approx (2/3)m_\mu^0/m_\tau^0 \approx 0.04$. Here $m_\mu^0 \simeq 3m_s^0$ has been used.

Another possible source for fermion masses corresponds to the non-renormalizable interactions with contributions of order

$$M_W \left(\frac{M_X}{M_{Pl}} \right)^n \lesssim \mathcal{O}(100 \text{ MeV}), \quad (2)$$

with the $n = 1$ contribution being of order the muon mass. One expects the relation $m_b^0 = m_\tau^0$ to hold at the 5% level in this case.

Consequently, in addition to allowing a 2σ error in $m_b(m_b)$, we also allow for deviation of 5% in the asymptotic relation $m_b^0 = m_\tau^0$, i.e.,

$$m_b^0 = m_\tau^0 [1 \pm 0.05] \quad (3)$$

In Fig. 1, we plot $m_b(m_b)$ versus $\tan\beta$ with $m_t = 150 \text{ GeV}$, $\alpha_s(M_Z) = 0.12$ and $M_S = m_t$. We have employed the appropriate two-loop renormalization group equation for the running of the gauge and Yukawa couplings between M_X and M_Z , while below M_Z we have used the 3-loop QCD[13] and one-loop QED β functions in the evolution of α_s, α and the mass parameters. The solid line in Fig. 1 corresponds to $m_b^0 = m_\tau^0$, while the dot-dashed lines

account for the $\pm 5\%$ deviation from the exact equality. It can be seen that there are two acceptable solutions for $\tan\beta$, corresponding to a given value of $m_b(m_b)$. The lower value is quite close to unity ($\tan\beta \simeq 1.1$ in this case), while the larger values exceed $\tan\beta = m_t/m_b$. Barring radiative corrections all intermediate values of $\tan\beta$ are disfavored! If we require $m_b(m_b) \lesssim 4.45 \text{ GeV}$, then $\tan\beta \lesssim 1.1$ or $\tan\beta \gtrsim 60$ are singled out.

Of course, all this assumes that radiative corrections to the b and τ masses can be ignored. It has been shown in Ref. [7] that this very much depends on a combination of parameters which include $\tan\beta$, the universal gaugino mass and the supersymmetric higgsino mass. In the case that we are interested in, to wit, $\tan\beta$ close to unity, we can a posteriori verify that the radiative correction can be kept small (\lesssim few %), without imposing stringent new constraints on the available parameter space.

The predictions for $m_b(m_b)$ are rather sensitive to the input values chosen for $\alpha_s(M_Z)$,

m_t and M_S . In Fig. 2, we plot the same curve, but with a lower value of 0.11 for $\alpha_s(M_Z)$. Now the allowed solutions are $\tan\beta \lesssim 1.1$ and $\tan\beta \gtrsim 42(\approx m_t/m_b)$. In Fig. 3, $\alpha_s(M_Z) = 0.12$ is kept, but $M_S = 1 \text{ TeV}$ is used. In this case, $\tan\beta \lesssim 1.1$ is the only allowed solution!

In Fig. 4, we display the variation of the prediction for $m_b(m_b)$ with the top-quark mass. Here, for clarity, $m_b^0 = m_\tau^0$ is the only case displayed, with m_t varying from 110 *GeV* to 190 *GeV*. Also $\alpha_s(M_Z) = 0.12$ and $M_S = 1 \text{ TeV}$ have been used. All intermediate values of $\tan\beta$ (say between 1.6 and 60) are excluded for $m_t \lesssim 170 \text{ GeV}$. In Fig. 5 we have plotted an enlarged version of Fig. 4 near $\tan\beta = 1$ for three values of m_t (150, 160 and 170 *GeV*) from which it is clear that $\tan\beta \lesssim 1.6$ at the lower end. However, for $m_t \gtrsim 180 \text{ GeV}$, all values of $\tan\beta$ are allowed. This feature is displayed in Fig. 6 where $m_t = 180 \text{ GeV}$ and $\alpha_s(M_Z) = 0.11$ are used.

Based on Figs. 1-6, the following conclusion can be reached. For $m_t \lesssim 170 \text{ GeV}$ and $\alpha_s(M_Z) \gtrsim 0.11$, and after allowing for uncertainties in the other relevant quantities, the two regions $\tan\beta$ close to unity, or $\tan\beta \gg 1$ are singled out.

It is an interesting coincidence that the asymptotic mass relation $m_b^0 \simeq m_\tau^0$ prefers the two extreme values of $\tan\beta$. Since the parameter space is narrow near the end points, a certain amount of fine-tuning is needed to obtain the desired values of $\tan\beta$. To see this explicitly, let us analyze the tree-level (neutral) higgs potential of the MSSM:

$$\begin{aligned} V_0 = & \mu_1^2 |H_1^0|^2 + \mu_2^2 |H_2^0|^2 + \mu_3^2 (H_1^0 H_2^0 + h.c.) \\ & + \frac{g_1^2 + g_2^2}{8} (|H_1^0|^2 - |H_2^0|^2)^2 \end{aligned} \quad (4)$$

where the mass parameters μ_i^2 are defined as

$$\mu_1^2 = m_{H_1}^2 + \mu^2; \quad \mu_2^2 = m_{H_2}^2 + \mu^2; \quad \mu_3^2 = B\mu \quad . \quad (5)$$

Here μ is the supersymmetric higgsino mass, and we let v_1, v_2 (both positive) denote the

two vevs, such that $v^2 = v_1^2 + v_2^2$ and $\tan\beta = v_2/v_1$. Minimization of Eq. (4) yields

$$\begin{aligned}\sin 2\beta &= \frac{2\mu_3^2}{\mu_1^2 + \mu_2^2} \\ v^2 &= \frac{2}{g_1^2 + g_2^2} \left[\frac{\mu_2^2 - \mu_1^2}{\cos 2\beta} - (\mu_1^2 + \mu_2^2) \right] .\end{aligned}\quad (6)$$

The large $\tan\beta$ solution would require $\mu_3^2 \ll (\mu_1^2 + \mu_2^2)$. The naturalness of this scenario has recently been discussed[14].

The scheme where $\tan\beta$ is close to unity requires a different fine-tuning condition. Let us take the limit of strict equality, $\tan\beta = 1$. In this case,

$$\mu_1^2 + \mu_2^2 - 2\mu_3^2 = 0 \quad (7)$$

from Eq. (6). Furthermore, $\mu_1^2 = \mu_2^2$ is needed to remedy the divergent behavior in v^2 (see Eq. (6)). In this limit, the Higgs potential has a custodial $SU(2)$ symmetry (under which H_1 and H_2 rotate into each other). This symmetry, of course, is broken badly in the Yukawa sector where $h_t \gg h_b$, as well as by the hypercharge gauge interactions. These effects show up in the Higgs sector only after one-loop radiative corrections are taken into account.

The $\tan\beta = 1$ solution preserves the custodial $SU(2)$ symmetry (at tree level), while the solution $\tan\beta \gg 1$ breaks this symmetry maximally. In the former case, the Yukawa interactions explicitly do not respect the custodial symmetry, whereas in the latter case they do. The hypercharge gauge interactions break this custodial symmetry in either case.

As a consequence of Eq. (6), the $\tan\beta = 1$ solution has a zero mass scalar at tree-level which can be seen as follows. The Higgs potential as a function of v^2 , after minimizing with respect to β , is given by

$$V(v^2)|_{\tan\beta=1} = v^2 \left[\frac{1}{2}(\mu_1^2 + \mu_2^2) - \mu_3^2 \right] . \quad (8)$$

The right-hand side of Eq. (8) vanishes as a consequence of Eq. (7), leaving v^2 undetermined. The resulting ambiguity along the radial direction gives rise to a (pseudo)

Goldstone mode. The relation (7) will be modified by the radiative corrections and is to be regarded as the starting point of the SUSY analogue of the Coleman-Weinberg mechanism[15]. This has been studied in some detail in the existing literature. The zero mass higgs mode, on account of the large top quark Yukawa coupling, as well as the existence of the scalar stops, can acquire a substantial mass through radiative corrections. Within the MSSM, with the large number of free parameters, the value of this mass can exceed 100 GeV . However, by embedding within a supergravity/GUT approach, we find this to be not the case.

It is important to ask if $\tan\beta \approx 1$ can be realized in a natural manner. One approach would be to replace the supersymmetric higgsino mass term by $\lambda N(H_1 H_2 - M^2)$, where N denotes a singlet superfield. The superpotential in this case possesses an R-symmetry under which all matter superfields as well as the superpotential change sign. Now the $SU(2) \times U(1)$ gauge symmetry can be broken at tree level whilst preserving supersymmetry. It is then “natural” to have $\tan\beta \simeq 1$, since we can now take the limit where all the SUSY breaking mass parameters are small compared to M^2 . Although intriguing, we will not pursue this possibility any further here.

Our analysis of radiative electroweak breaking will be based on the renormalization group improved tree level scalar potential given in Eq. (4). The various parameters evolve according to the well known one loop RGE[16] from M_X to the low energy scale Q_0 . A judicious choice of Q_0 [17] will yield a fairly reliable estimate of the parameters, including bounds on the sparticle mass spectrum.

The spontaneous breaking of the electroweak gauge symmetry imposes the constraint

$$\mu_1^2 \mu_2^2 \leq \mu_3^4 \tag{9}$$

while the boundedness of the scalar potential yields

$$\mu_1^2 + \mu_2^2 \geq 2|\mu_3|^2 \quad . \tag{10}$$

The previous considerations of $m_b(m_b)$ help us pin down the range of values at M_X for the Yukawa couplings h_b, h_τ and h_t . The parameter $\tan\beta$ at scale $Q_0 \sim M_S$ lies between 1 and 1.6. In addition, we have the parameters $M_{\frac{1}{2}}, m_0, A$ at M_X , which denote the common gaugino mass, the common scalar mass and the common trilinear scalar coupling. Once these are specified, the one loop beta functions are integrated to yield the gaugino and scalar mass parameters, as well as the trilinear couplings at Q_0 . Following Ref. [17], we impose the constraint

$$|A(M_X)| < 3 \quad (11)$$

Note that the structure of the one loop beta functions does not require us to know the values of the parameters μ (the SUSY higgsino mass) and the bilinear coupling parameter $B(\equiv \mu_3^2/\mu)$ at this stage of the computation. Indeed, once $\tan\beta$ is known from the considerations described earlier, we obtain from the minimization conditions:

$$\mu^2(Q_0) = \left[\frac{1}{2} M_Z^2 (\tan^2 \beta - 1) - (m_{H_1}^2 - \tan^2 \beta m_{H_2}^2) \right] / (1 - \tan^2 \beta) . \quad (12)$$

Further it can be seen that

$$\mu_3^2(Q_0) = \tan \beta \left[-m_{H_2}^2 - \mu^2 - \frac{1}{2} M_Z^2 \left(\frac{\tan^2 \beta - 1}{\tan^2 \beta + 1} \right) \right] . \quad (13)$$

There is a two fold ambiguity in the physical spectrum for a given point in the parameter space due to the fact that μ can be of either sign. This is found to affect the physical spectrum significantly because of the structure of the terms in the sfermion, neutralino and chargino mixing matrices when the electro-weak symmetry is broken. We find that the dependence of the spectrum on A is minimal and therefore stick to the choice $A = 0$ for the most part. The beta functions are such that the physical spectrum remains unaltered under the interchange $(-M, A)$ with $(M, -A)$. Thus, with $A = 0$ the physical spectrum shows no dependence on the sign of M .

Armed with this a systematic search is performed to find the acceptable region of the parameter space and the associated sparticle spectrum. It may be seen that with all other parameters fixed, as $\tan\beta$ approaches 1, the parameter μ rises. An unusually large $|\mu|$ may be unattractive, and so we impose a somewhat ad hoc upper bound of 1 TeV on $|\mu|$ (radiative corrections to the b-mass can become significant if $|\mu|$ is excessively large). This establishes a lower bound on $\tan\beta$ (separately for the choice of positive or negative μ). For a given $\tan\beta$ there is a unique value of the common gaugino and scalar mass for which the lightest neutralino is the LSP (albeit degenerate with the lighter stau and stop). This follows from the following observations. As m_0 is increased, with all other parameters fixed, μ increases (thus an upper bound on m_0 emerges naturally). For a sufficiently small $\tan\beta$, as m_0 is increased, the mass of the lighter scalar top falls, due to the mixing term increasing in importance. However, a sufficiently large m_0 is required to ensure that the lighter scalar tau is heavier than the lightest neutralino (the would be LSP). As m_0 increases, the bino-purity of the lightest neutralino is also found to increase. We impose a constraint that the bino-purity of the lightest neutralino be $\gtrsim 95\%$ [18] in order to ensure that the LSP is a good candidate for cold dark matter.

The above considerations are used in determining the upper and lower bounds on the two primary soft SUSY parameters $M_{\frac{1}{2}}$ and m_0 . It is found that as $\tan\beta$ migrates away from its lower bound the parameter space opens up somewhat. This is because as $\tan\beta$ increases, even with fairly large values of $M_{\frac{1}{2}}$ and m_0 , we meet all of the consistency conditions, with $|\mu|$ staying considerably smaller than a 1 TeV .

We illustrate in the figures the behavior of the parameters of the theory. In Figs. 7a, 7b we exhibit the upper and lower bounds on $M_{\frac{1}{2}}$ (and the corresponding m_0 values) as functions of $\tan\beta$. [Note that without the restriction $|\mu| \lesssim 1\text{ TeV}$, the upper bound on $M_{\frac{1}{2}} \approx 800\text{ GeV}$, corresponding to a bino mass of $\approx 350\text{ GeV}$.] Due to the fact that for a given point in the parameter space the squark mixing is less important when μ is negative, the lower bounds emerge smaller. In Figs. 8a, 8b we show, for a particular choice of some of the parameters, the variations of the lighter stau and stop masses as

m_0 is varied. In Fig. 9 we show the behavior for a larger value of $\tan\beta$. Comparing Figs. 8 and 9 we find that the parameter space is much more constrained in the former case. Finally, Figs. 10a, 10b display how the bino purity varies with m_0 , while Fig. 11 shows the variation of $|\mu|$ with m_0 for fixed $M_{\frac{1}{2}}$.

To explain the shapes a brief discussion is in order. At the smallest value of $\tan\beta$ realizable, we meet the conditions that $|\mu| \approx 1 \text{ TeV}$ and that the lighter stop and stau are degenerate with the lightest neutralino. As $\tan\beta$ increases, we can find smaller values of the unified gaugino mass such that the lighter stop and stau continue to be degenerate with the lightest neutralino, with $|\mu| < 1 \text{ TeV}$. However, with increasing $\tan\beta$ the lightest neutralino begins to mix strongly with one of the higgsinos. We require that the bino purity remain in excess of, and restrict the parameter range further by requiring that the $< 95\%$ lighter stau mass be no larger than three times the LSP mass. The maximum value of $M_{\frac{1}{2}}$ for a given $\tan\beta$ is reached when m_0 is chosen such that the lightest neutralino is the LSP, and simultaneously the upper bound on $|\mu|$ is saturated. In Fig. 8, 9, the shapes of the stau and stop mass contours are easily understood. Since the mixing term is negligible for the stau, increasing m_0 leads to increasing values of the stau. In the stop sector, however, as m_0 increases, $|\mu|$ increases leading to appreciable mixing and the diminishing of the smaller eigenvalue of the stop mixing matrix, i.e., the mass of the lighter physical stop. Indeed as discussed earlier, as $\tan\beta$ falls, this mixing becomes even more pronounced for a given point in the parameter space, and thus the parameter space becomes more constrained since the stop must remain lighter than the lightest neutralino.

It has been argued and found to be true by comparing our computations with other related studies that the tree level potential is consistent to within 10% with the complete one loop scalar potential. This is achieved by the selection of Q_0 to be of the order of the geometric mean of the scalar top masses. We also study the variations in the spectrum for Q_0 between the two extreme limits of M_Z and 1 TeV .

As far as the sparticles are concerned it is not inconceivable that the lighter stau

and stop will be found at LEP 200. Equally likely, however, is the possibility that the sparticles are all heavy, accessible perhaps only at the LHC. This is the case where the bino LSP has mass $\sim 350 \text{ GeV}$, one of the staus is the lightest charged sparticle, while the colored sparticles approach and even exceed the TeV range.

As far as the higgs sector is concerned, with $m_t(m_t) \lesssim 170 \text{ GeV}$, $|\mu| < 1 \text{ TeV}$, stop masses $\lesssim 1 - 2 \text{ TeV}$, and $A_t(Q_0) \lesssim 1 \text{ TeV}$, the lightest (CP even) higgs mass is expected to be between 60 GeV and 100 GeV [19]. It offers the best prospects for a truly remarkable discovery at LEP 200. The CP odd scalar mass exceeds 300 GeV which implies that the remaining scalar higgs will not be accessible for quite some time.

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Figure Captions

Fig. 1. Plot of $m_b(m_b)$ vs. $\tan\beta$ for $\alpha_S(M_Z) = 0.12$.

Fig. 2. Plot of $m_b(m_b)$ vs. $\tan\beta$ for $\alpha_S(M_Z) = 0.11$.

Fig. 3. Plot of $m_b(m_b)$ vs. $\tan\beta$ for a SUSY threshold of 1 TeV.

Fig. 4. Contours of $m_b(m_b)$ vs. $\tan\beta$ for $m_t(m_t)$ varying from 110 GeV to 190 GeV.

Fig. 5. Inset of Fig. 4 near $\tan\beta = 1$.

Fig. 6. Plot of $m_b(m_b)$ vs. $\tan\beta$ for $m_t(m_t) = 180$ GeV, $\alpha_S = 0.11$.

Fig. 7a. Plots of mass parameters $M_{\frac{1}{2}}$ and m_0 vs. $\tan\beta$ for $\mu > 0$.

Fig. 7b. Same as in Fig. 7a with $\mu < 0$.

Fig. 8a. Plot of bino, stop and stau mass vs. m_0 for a choice of parameters with $\mu > 0$.

Fig. 8b. Same as in Fig. 8a with $\mu < 0$.

Fig. 9a. Same as in Fig. 8a with a larger $\tan\beta$ value .

Fig. 9b. Same as in Fig. 8b with a larger $\tan\beta$ value.

Fig. 10a. Bino purity vs. m_0 for a choice of parameters with $\mu > 0$.

Fig. 10b. Same as in Fig. 10a with $\mu < 0$.

Fig. 11. Plot of $|\mu|$ vs. m_0 for a choice of parameters.